

INDIAN STATISTICAL INSTITUTE, Bangalore Centre
Midsem
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Course Name: Complex Analysis

Duration: 2 hours

1. Let $\Omega \subseteq \mathbb{C}$ be a domain.

[a] Let h and k be two harmonic functions on Ω such that $h \equiv k$ on a non-empty open set $A \subseteq \Omega$. Then prove that $h \equiv k$ on Ω .

[b] Let h be a harmonic function defined on Ω which attains local maxima in Ω , then h is identically constant on Ω . [5+5]

2. Determine whether the following statements are TRUE or FALSE with justification.

[a] Let $a_j > 0$, for $j = 1, 2, \dots$. If $\sum_{j=0}^{\infty} a_j z^j$ is convergent on $D(0; r) = \{z \in \mathbb{C} : |z| < r\}$ and if $\epsilon > 0$ is sufficiently small, then $\sum_{j=0}^{\infty} (a_j + \epsilon) z^j$ is convergent on $D(0; r')$ for some $0 < r' < r$.

[b] Let f be an entire function, then $f(\mathbb{C})$ is dense in \mathbb{C} . [5+5]

3. Let Ω be a bounded domain in \mathbb{C} , and let $\varphi : \Omega \rightarrow \Omega$ be an analytic function. If there exists a point $z_0 \in \Omega$ such that $\varphi(z_0) = z_0$ and $\varphi'(z_0) = 1$, then prove that φ is a linear map, i.e., of the form $z \rightarrow az + b$, for some $a, b \in \mathbb{C}$. [10]